

Exercise 1C ("logistic DE")

Consider the logistic DE from population dynamics:

$$\begin{cases} y' = y(1-y) & , t \in (0, T] \\ y(0) = y_0. \end{cases}$$

- determine the analytic solution of this DE.
- plot the solution $y(t)$ for several initial conditions y_0 . Consider $y_0 > 1$, $0 < y_0 < 1$ and $y_0 < 0$.
- Approximate this DE with the methods CT (central in time), CTnl (central in time, nonlocal), FT (forward in time), FTnl (forward in time, nonlocal) and Runge-Kutta 2. The terms "nonlocal" and these methods are explained in the extra document on the webpage.
- Find the exact expressions for y_n in terms of y_0 , Δt and the index n , for the methods CTnl and FTnl.
- Perform numerical experiments with Matlab. In particular, choose for CT: $y_0 = 0.5$, $\Delta t = 0.1$ and $T = 200$; for CTnl: $y_0 = 0.25$ (choose Δt and T yourself); for FT: $y_0 = 0.5$ with $\Delta t = 0.01$ ($T = 10$), $\Delta t = 1.5$ ($T = 45$) and $\Delta t = 2.5$ ($T = 50$), for FTnl: $y_0 = 0.5$ with $\Delta t = 0.01$ ($T = 10$), $\Delta t = 1.5$ ($T = 45$), $\Delta t = 2.5$ ($T = 50$) and $\Delta t = 3$ ($T = 60$).

What kind of numerical effects do you observe?

Can you explain them?

Do you find chaotic solutions?