Home exercise

τ – model from geo-hydrology

$$u_t = d\,u_{xx} + [f(u)]_x + \tau\,u_{xxt}$$

$$u(0,t) = u_-,\quad u(1.4,t) = u_+ \qquad x \in [0, 1.4],\ t \in [0, T]$$

$$u(x,0) = u_- + \frac{1}{2}(u_+ - u_-)(1 + \tanh(R(x - x_0)))$$

Take: $u_- = 0$, $u_+ = 0.6$, $R = 50$, $x_0 = 1.2$

Apply the first step in the Method-of-Lines:

$$\dot{\vec{u}}(t) = d D_2 \vec{u}(t) + \vec{q}(\vec{u}) + \tau D_2 \dot{\vec{u}}(t)$$

In the second step (after re-arranging the terms): IMEX

Choose: $\Delta x = 0.005$ and $\Delta t = \ldots$?

1) "one-phase flow" $f(u) = \frac{u^2}{2}$, $d = 10^{-3}$, $\tau = 3 \cdot 10^{-4}$, $T = 2$

2) "two-phase flow" $f(u) = \frac{u^2}{u^2 + (1-u)^2}$, $d = 10^{-3}$, $\tau = 3 \cdot 10^{-4}$, $T = 1$

3) "no overshoot" $f(u) = \frac{u^2}{2}$, $d = 10^{-3}$, $\tau = 0$, $T = 2$