mastermath course Symplectic Geometry

Assignment 13

The exercises marked with a $*$ are **inleveropdrachten**.

The exercises marked with a $+$ are particularly important.

Exercise 1 (coisotropic submanifolds are presymplectic) *Let (M, ω) be a symplectic manifold and $N \subseteq M$ a coisotropic submanifold. We denote by*

$$i : N \to M$$

the inclusion map. Prove that $(N, i^*\omega)$ is a presymplectic manifold.

The following result is used in the proof of the theorem about symplectic reduction in the lecture.

*** Exercise 2 (isotropic distribution involutive)** *Let (M, ω) be a presymplectic manifold. Prove that the isotropic distribution TM^ω is involutive, i.e., if X and Y are vector fields on M that take values in TM^ω then their Lie bracket also takes values in this distribution.*

Exercise 3 (equivalence relation defined by distribution) *Let M be a manifold and $E \subseteq TM$ a distribution. Show that the set*

$$R^E := \big\{ \big(x(0), x(1)\big) \,\big|\, x \in C^\infty\big([0,1], M\big) : \dot{x}(t) \in E_{x(t)}, \forall t \in [0,1] \big\}$$

is an equivalence relation on M.

The following exercise was used in the lecture in the proof of a theorem (symplectic reduction).

Exercise 4 (kernel of the differential of the canonical projection) *Let M be a manifold and E an integrable distribution on M. Assume that the corresponding foliation is regular, i.e., there exists a manifold structure on the set of leaves $\overline{M}$, for which the canonical projection $\pi : M \to \overline{M}$ is a submersion. Show that*

$$\ker d\pi(x) = E_x, \quad \forall x \in M.$$

By an exercise in Assignment 12 (smooth structure unique) the manifold structure on $\overline{M}$ is unique. The differential $d\pi(x)$ is with respect to this unique structure.

Hint: *Prove and use the following:*

Lemma 1 *Let M be a manifold and E an integrable distribution on M. Let F be a leaf of the corresponding foliation on M. There exists a manifold structure of dimension* rank$E$ *on F such that the inclusion $F \to M$ is an immersion.*